٢٠١٧ /١٣/١ lec 4 د/خسرو صمد

* Electric and Magnetic Potential vectors $\bar{V}$ and $\bar{A}$

① Static Case (Not Time Varying)

أولاً for electric Potential vector $\bar{V}$

* The relation between the electric field $\bar{E}$ and the electric Potential vector $\bar{V}$ is given by

① * $\bar{E} = -\nabla \bar{V} \Rightarrow \frac{\partial V}{\partial x}\hat{x} + \frac{\partial \bar{V}}{\partial y}\hat{y} + \frac{\partial \bar{V}}{\partial z}\hat{z}$

* $\bar{D} = \epsilon \bar{E}$

* $\nabla \cdot \bar{D} = \rho_v \rightarrow$ Source

② * $\nabla \cdot \bar{E} = \frac{\rho_v}{\epsilon}$

بالتعويض من ① و ②

* $\nabla \cdot (-\nabla V) = \frac{\rho_v}{\epsilon}$

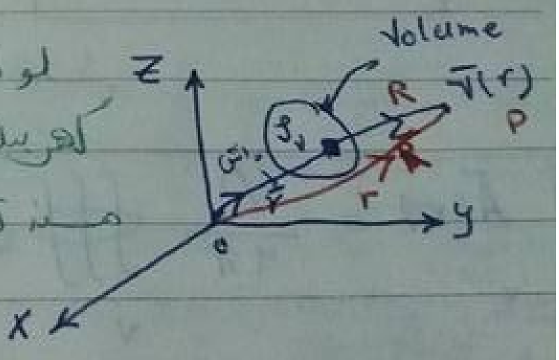
③ * $\boxed{-\nabla^2 \bar{V} = \frac{\rho_v}{\epsilon}} \Rightarrow$ Poisson's equation *

حلها.

* $\bar{V}(r) = \frac{1}{4\pi\epsilon} \iiint_V \frac{\rho_v(\bar{r}')}{R}\, dv \Rightarrow$ Ⓐ معادلة الجهد الكهربى

(داخلي)

لو فرضنا وجود Volume موزع عليه شحنة كهربية مقدارها ρ_v وموجود على مسافة r' من نقطة الأصل فانه سوف يولد جهد كهربى مقداره $\bar{V}(r)$ على مسافة (R) من الجسم وعلى مسافة (r) من نقطة الأصل



$\Rightarrow \rho_v \rightarrow V(r) \rightarrow \bar{E} = -\nabla \bar{V}$

ماكسويل

$\bar{H}$

P

* Magnetic Potential vector $\bar{A}$

* ① $\nabla \cdot \bar{B} = 0$

* ② $\nabla \cdot (\nabla \times \bar{A}) = 0$

بمقارنة ① و ②

$$\text{so } \bar{B} = \nabla \times \bar{A}$$

$$\bar{B} = \mu \bar{H}$$

* ③ $\bar{H} = \frac{1}{\mu}(\nabla \times \bar{A})$

④ $\leftarrow$ $\nabla \times \bar{H} = \bar{J} \rightarrow$ Source

بأخذ ③ ∇

$$\nabla \times \bar{H} = \bar{J} = \frac{1}{\mu}(\nabla \times \nabla \times \bar{A})$$

$$\frac{1}{\mu}(\nabla \times \nabla \times \bar{A}) = \bar{J}$$

الحد يعطي كمية قياسية

$$\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu \bar{J}$$

كمية قياسية تفاضل

=0

$$\boxed{-\nabla^2 \bar{A} = \mu \bar{J}}$$

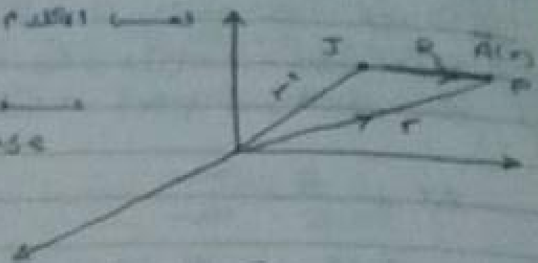
معادلة بواسون

الحل.

$$\boxed{\bar{A}(r) = \frac{\mu}{4\pi} \iiint_v \frac{\bar{J}(\bar{r})}{R}\, dv}$$

[illegible] الكلام [illegible]

Magnetic Potential
Drive expretion for
A , $\bar{V}$ & static case
* Time varying case



$$\bar{J}(r') \xrightarrow{\int} \bar{A}(r) \xRightarrow{\frac{d}{dr}} \bar{H} = \frac{1}{\mu}(\nabla \times \bar{A})$$

* Time Varying Case *

$$\bar{E}(r,t) = \underbrace{\bar{E}(r)}_{\text{Mag.}} \underbrace{e^{J\omega t}}_{\text{phase}}$$

$$\bar{H}(r,t) = \bar{H}(r) \cdot e^{J\omega t}$$

$$\rho_v(r,t) = \rho_v(r) e^{J\omega t}$$

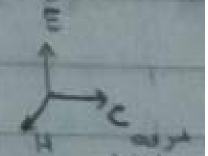
$$\bar{J}(r,t) = \bar{J}(r) e^{J\omega t}$$

$$\bar{A}(r,t) = A(r) e^{J\omega t}$$

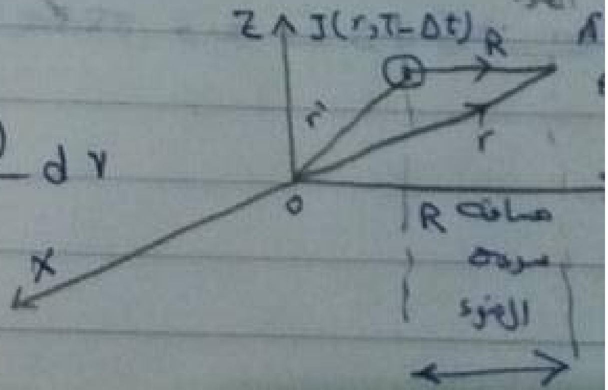
$$\bar{V}(r,t) = V(r) e^{J\omega t}$$

Case ① $\bar{A}$

$$\Rightarrow \bar{A}(r,t) = \frac{\mu}{4\pi} \iiint_V \cancel{\frac{\bar{J}(r',t)}{R}} \, dv$$



$$\bar{A}(r,t) = \frac{\mu}{4\pi} \iiint_V \frac{\bar{J}(r', t-\Delta t)}{R} \, dv$$



Drive expression A & V in Time vary

* $\bar{A}(r,t)$: $\bar{A}$ عند اللحظة (t) ناتج عن تيار (J) ولد موجة كهرومغناطيسية عند اللحظة $(t-\Delta t)$ ما قطعت مسافة (R) بسرعة الضوء حتى وصلت عند اللحظة (t)

$$\Delta t = \frac{R}{C} \quad \frac{\text{مسافة}}{\text{سرعة}}$$

$$\bar{A}(r)\, e^{J\omega t} = \frac{\mu}{4\pi} \iiint_V \frac{\bar{J}(r')\, e^{J\omega t}\, e^{-J\omega \Delta t}}{R}\, dv$$

$$\bar{A}(r) = \frac{\mu}{4\pi} \iiint_V J(r') \frac{e^{-J\omega \Delta t}}{R}\, dv$$

$$\omega \Delta t = \omega \frac{R}{c}$$

$$= \frac{2\pi f}{c} R = \overset{B}{\frac{2\pi}{\lambda}} R \qquad B = \frac{2\pi}{\lambda}$$

$$= BR$$

$$\bar{A}(r) = \frac{\mu}{4\pi} \iiint_V J(r') \frac{e^{-JBR}}{R}\, dv$$

نبحث

$$* \quad \bar{V}(r) = \frac{1}{4\pi \epsilon} \iiint_V \rho_v(r') \frac{e^{-JBR}}{R}\, dv$$

لو طلب الاثبات في المعادلات دي #

① نبدأ بالاثبات من اول STATIC case وبعد كده ندخل على Time varying ونستبدل

[1] Dipole antennas

* are wire antennas That can be classified according To Their lengThs, They are classified into 3 ثلاث CaTegories :

① ElemenTary dipole (infiniTisimal) $\Delta L < \frac{\lambda}{100}$

متناهي الصغر

[2] ShorT dipole $\frac{\lambda}{100} < \Delta L < \frac{\lambda}{10}$

[3] long dipole $L > \frac{\lambda}{10}$

طول ال antenna

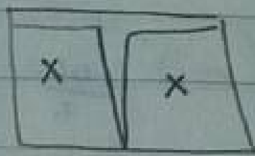
Narrow band a

* ملحوظة : في حل اى مسائل لازم التأكد من طول اى antenna لتحديد نوعها

[1] ElemenTary dipole

① $\Delta L < \frac{\lambda}{100}$

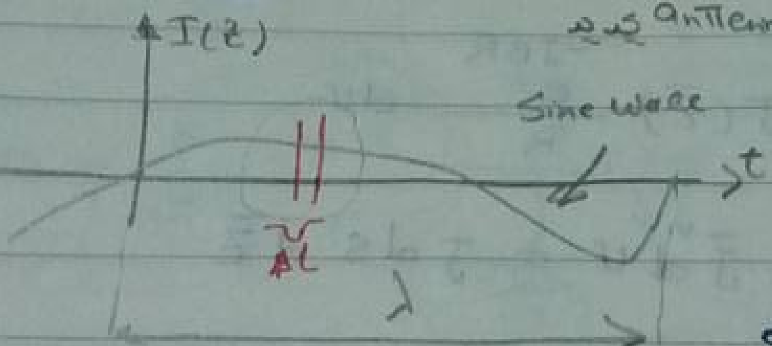
$S_{11} = \Gamma$

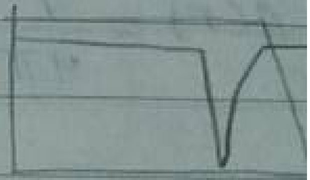


ملحوظ

طول ال antenna يحدد

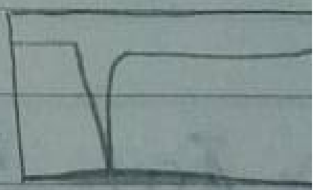
[2]





ملحوظة

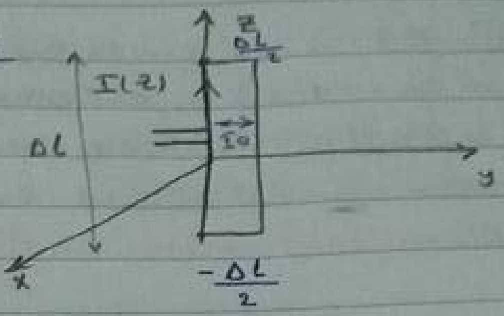
* طول ال antenna هو الذى يحدد ال Current distribuTion



*② The CurrenT disTribuTion accross The antenna is ConsTanT.

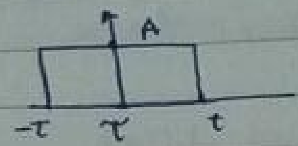
resonant circuit

$$I(z) = \begin{cases} I_0 & -\frac{\Delta L}{2} \le z \le \frac{\Delta L}{2} \\ 0 & \text{o.w} \end{cases}$$



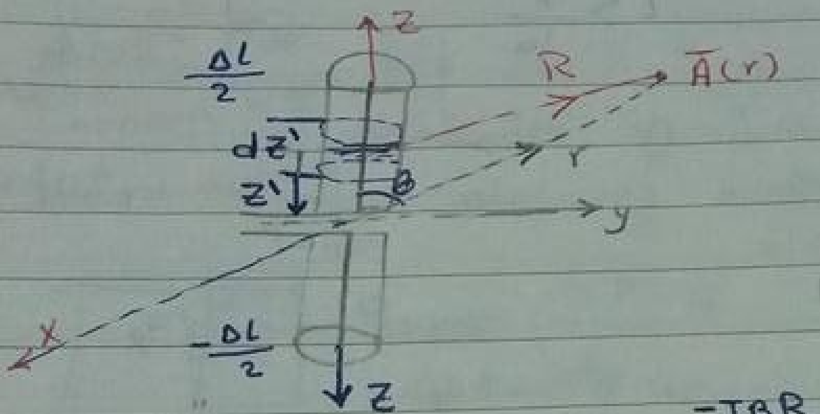
rect. دالة

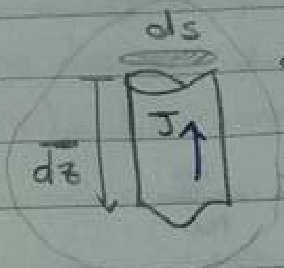
$$\boxed{I(z) = I_0 \, \text{rect} \, \frac{z}{\Delta L}} \quad \#$$



$A \, \text{rect} \left(\frac{t}{\tau} \right)$

* Derive an expression for $\bar{A}$





$dv = ds \, dz'$

$$\bar{A}(r) = \frac{\mu}{4\pi} \iiint_V J(\bar{z}) \frac{e^{-j\beta R}}{R} \, dv$$

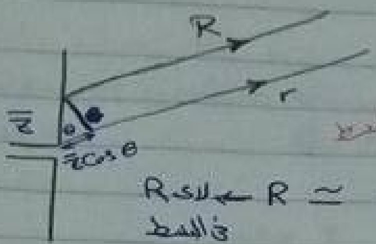
$$\bar{J} \, dv = J \, ds \, d\bar{z}$$

$$= I(z') \, d\bar{z}$$

$$\bar{A}(r) = \frac{\mu}{4\pi} \int_{-\frac{\Delta L}{2}}^{\frac{\Delta L}{2}} I(z') \frac{e^{-j\beta R}}{R} \, dz'$$

* at far field

* approximation



الشكل $\underline{\underline{R}}$ موازي الخط
في المقام

R في الأس $\leftarrow$ $R \simeq r - \bar{z}\cos\theta \longrightarrow (1)$
في الأس

في المقام لا R في المقام $\quad \dfrac{1}{R} \simeq \dfrac{1}{R} \longrightarrow (2)$

$$\bar{A}(r) = \frac{M}{4\pi}\int_{-\frac{\Delta l}{2}}^{\frac{\Delta l}{2}} I(z')\,\frac{e^{-JB(r - z'\cos\theta)}}{r}\,d\bar{z}$$

$$\bar{A}(r) = \frac{M}{4\pi}\,\frac{e^{-JBr}}{r}\int_{-\frac{\Delta l}{2}}^{\frac{\Delta l}{2}} I(\bar{z})\,e^{J(B\bar{z}\cos\theta)z'}\,dz'$$

let $\boxed{B\cos\theta = \omega}$ $\qquad f(\omega) = \int_{-\infty}^{\infty} f(t)\,e^{J\omega t}\,dt$

$$\bar{A}(r) = \underbrace{\frac{M}{4\pi}\,\frac{e^{-JBr}}{r}}_{\text{ثابت}}\;\underbrace{\int_{-\frac{\Delta l}{2}}^{\frac{\Delta l}{2}} I(z')\,e^{J\omega\bar{z}}\,dz'}_{F.T}$$

تقريبا يكافئ F.T of $I(z')$

$$\bar{A}(r) = \frac{M}{4\pi}\,\frac{e^{-JBr}}{r}\,F.T\left[I(z')\right]$$

$I_o \,\mathrm{recT}\left(\dfrac{z}{\Delta l}\right) \xrightarrow{F.T} I_o\,\Delta l\,Sa\left(\dfrac{\omega\Delta l}{2}\right)$

$$\bar{A}(r) = \frac{M}{4\pi}\,\frac{e^{-JBr}}{r}\,I_o\,\Delta l\,Sa\left(\frac{\omega\Delta l}{2}\right) \longrightarrow (1)$$

$Sa\left(\frac{\omega \Delta l}{2}\right)$

$$\frac{\omega \Delta l}{2} = \frac{B \cos\theta \, \Delta l}{2}$$

$$= \frac{2\pi}{\lambda} \frac{\Delta l}{2} \cos\theta$$

$$= \frac{\pi}{\lambda} \frac{\lambda}{100} \underbrace{\cos\theta}_{max=1} = \frac{\pi}{100} = 0.0314 \cong 0$$

$Sa(0) = 1$

$$\bar{A}(r) = \frac{\mu}{4\pi} \frac{e^{-jBr}}{r} I_o \Delta l$$

#

Drive ~~expretion~~ expression سؤال